Section 9.3

Base b Logarithm
The base b logarithm of x, is the power to which we need to raise b in order to get x. Symbolically,

$$\log_b x = y \qquad \text{means} \qquad b^y = x.$$

Common logarithm and natural logarithm:

$$\text{Base 10: } \log_{10} x = \log x \qquad \text{Common Logarithm}$$
$$\text{Base } e\text{: } \log_e x = \ln x \qquad \text{Natural Logarithm}$$

Change of Base Formula

$$\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$$

Logarithmic Function
A logarithmic function has the form

$$f(x) = \log_b x + C \qquad \text{or, alternatively,} \qquad f(x) = A \ln x + C$$

Logarithmic Identities
The following identities hold for all bases $a \neq 1$ and $b \neq 1$, all positive numbers x and y, and every real number r. These identities follow from the laws of exponents.

a) $\log_b (xy) = \log_b x + \log_b y$
b) $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
c) $\log_b (x^r) = r \log_b x$
d) $\log_b b = 1;\ \log_b 1 = 0$
e) $\log_b x = \frac{\log_a x}{\log_a b}$
f) $\log_b (b^x) = x$
g) $b^{\log_b x} = x$

Exponential Decay Model and Half-Life
And exponential decay function has the form

$$Q(t) = Q_0 e^{-kt}$$

Q_0 represents the value of Q at time $t = 0$, and k is the decay constant.

Exponential Growth Model and Doubling Time
An exponential growth function has the form

$$Q(t) = Q_0 e^{kt}$$

Q_0 represents the value of Q at time $t = 0$, and k is the growth constant.

Problem 1. Write the following values in the logarithmic form.

$$4^3 = 64, \qquad 10^{-1} = 0.1, \qquad 2^8 = 256, \qquad 5^0 = 1, \qquad (0.5)^2 = 0.25, \qquad 6^{-2} = \frac{1}{36}$$

Problem 2. Write the following logarithms in the exponential form.

$$\log_5 5 = 1, \qquad \log_4 \frac{1}{16} = -2, \qquad \log_4 16 = 2, \qquad \log_{10} 10{,}000 = 4, \qquad \log_7 1 = 0$$

Problem 3. Solve the following equations.

a) $6^{3x+1} = 30$

b) $4(1.5^{2x-1}) = 8$

Problem 4. Find the associated exponential growth or decay model.

a) $Q = 2000$ when $t = 0$; Half-life $= 5$.

b) $Q = 2000$ when $t = 0$; Doubling-time $= 4$.

Problem 5. Convert the given exponential function to the form indicated.

a) $f(x) = 2.1e^{-0.1x}$; $f(x) = Ab^x$

b) $f(t) = 23.4(0.991)^t$; $f(t) = Q_0 e^{-kt}$

Problem 6. Plutonium-239 is used as a fuel for some nuclear reactors, and also as a fissionable material in atomic bombs. It has a half-life of 24,400 years. How long would it take 10 grams of plutonium-239 to decay to 1 gram?

Problem 7. The half-life of strontium-90 is 28 years.

a) Obtain an exponential decay model for strontium-90 in the form $Q(t) = Q_0 e^{kt}$.

b) Use the model to predict, to the nearest year, the time it takes three-fifths of a sample of strontium-90 to decay.

Problem 8. The rate of auto thefts is doubling every four months.

a) Determine the base b for an exponential model $y = Ab^t$ of the rate of television thefts as a function of time in months.

b) Find the tripling time to the nearest tenth of a month.

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz: #1, 3, 10, 19, 22, 23, 31, 33, 41, 49, 51, 53